

SyZyGy: A Straight Interferometric Spacecraft System for Gravity Wave Observations

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(Dated: February 7, 2008)

Abstract

We consider a spaceborne gravitational wave (GW) detector formed by three spacecraft in a linear array, coherently exchanging laser beams and using the data combinations of time-delay interferometry (TDI). We previously showed how the measured time series of Doppler shifts in the six one-way laser links between spacecraft pairs in a general unequal-arm triangular configuration can be combined, using TDI, to exactly cancel the otherwise overwhelming phase noise of the lasers while retaining sensitivity to GWs. Here we apply TDI, unfolding the general triangular configuration, to the special case of a linear array of three spacecraft. We show that such an array (“SyZyGy”) has, compared with an equilateral triangle GW detector of the same scale, degraded (but non-zero) sensitivity at low-frequencies ($f \ll c/(\text{array size})$) but similar peak and high-frequency sensitivities to GWs. We develop the GW and noise responses, noting that in this geometrical case only one TDI combination is GW-sensitive showing a relatively simple 6-pulse response to an incident GW burst. Sensitivity curves are presented for SyZyGys having various arm-lengths. A number of technical simplifications result from the linear configuration. These include: only one faceted (e.g. cubical) proof mass per spacecraft, intra-spacecraft laser metrology needed only at the central spacecraft, placement in an appropriate orbit can reduce Doppler drifts so that no laser beam modulation is required for ultra-stable oscillator noise calibration, and little or no time-dependent articulation of the telescopes to maintain pointing. Because SyZyGy’s sensitivity falls off more sharply at low-frequency than that of an equilateral triangular array, it may be most useful for GW observations in the band between those of ground-based interferometers ($\sim 10 - 2000$ Hz) and LISA ($\sim 10^{-4} - 10^{-1}$ Hz). SyZyGy with ~ 1 light-second scale could, for the same instrumental assumptions as LISA, make observations in this intermediate frequency GW band with 5σ sensitivity to sinusoidal waves $\simeq 2.5 \times 10^{-23}$ in a year’s integration.

PACS numbers: 04.80.N, 95.55.Y, 07.60.L

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I. INTRODUCTION

When coherent optical or microwave wave trains are interchanged between three or more freely orbiting spacecraft, and phase or frequency differences of all incoming and outgoing beams recorded, there is a closure that allows subsequent elimination of all source fluctuations (“laser noises”) and so achieves sensitivity to propagating gravitational waves at much lower levels set by secondary noises. In doing this, the precise propagation times between sources and readout locations enter the data reduction algorithms, but equality of arm lengths is not necessary. We have called this scheme “time delay interferometry” or TDI, [1] and subsequently analyzed several configurations and data types needed for further elimination of the most serious secondary phase noise sources, *viz.* those due to spacecraft non-geodesic motion (“optical bench noise”) and especially to fluctuations of the ultra-stable local oscillators, or frequency standards, used in fringe tracking (“USO noise”) [2, 5]. Fringe tracking is necessary when the lasers in the configuration have widely differing frequencies, and/or when the configuration does not move rigidly so that emitted and received signals are Doppler shifted during slow spacecraft separation changes.

The LISA space gravitational wave antenna will use TDI, and is to be roughly equilateral, which maximizes the overall response to low frequency gravitational waves over a wide band. Reduction of secondary noise, and simplicity of system architecture, nevertheless suggest consideration also of other schemes; here we consider the most extreme possible application of TDI, *viz.* to a linear orbiting array, or, in astronomical terms, three spacecraft flying in syzygy¹.

We have previously considered a linear array, or parallel beam interferometric detector of gravitational waves, in work showing that with multiple bounces a split antiparallel laser beam will amplify a gravitational wave signal while laser fluctuations are cancelled conventionally by precise control of equal arm lengths [6]. We now see the multiple readout scheme of TDI as much preferable, and equal spacing of the three spacecraft is no longer required. The equations for combining the heterodyne data streams in TDI can be taken directly from

¹ An amusing coincidence is that Dhurandhar, Nayak and Vinet [6] have pointed out that underlying the data combinations of TDI is the mathematics of polynomial rings in three variables, and that the structure we found is called by algebraists a “module of syzygies”! SyZyGy for this reason again would seem to be an apt acronym for the concept of this paper.

our previous papers, which had general triangular configuration; in this paper they will be used in the linear limit.

The (nearly) linear formation of SyZyGy has some advantages and disadvantages over the nominal equilateral triangular formation planned for the LISA mission. The primary disadvantages of the linear formation are less sensitivity at the low frequencies (for a given spacecraft separation distance) and sensitivity to only one linear combination of GW polarizations. The advantages arise from the potential for a more stable formation geometry. The nominal LISA triangular formation involves spacecraft in orbits with semi-major axis a , inclination I , and eccentricity e related by $e = I/3^{1/2}$ and $I = L/(2a)$ [7, 8]. This choice of orbital elements produces a rotating triangular configuration with arm lengths that are constant to $\sim e^2 L$. The time variation of the arm lengths causes a variable Doppler shift on the laser beam signals, a variable direction from each spacecraft to the other two spacecraft, and a variable angle between the spacecraft velocity and the direction to the other spacecraft. The (almost) linear array might be formed with three spacecraft moving in a single circular orbit about the sun. In the ideal case (ignoring perturbations from other bodies) the distance between spacecraft and the directions from each spacecraft to the others will remain constant. The constant distance between spacecraft removes the Doppler shift on the laser signals. The constant directions means that no mechanism is needed to account for angle variation. The constant angle also means that a single faceted (e.g. cubical) test mass can be used in each spacecraft, instead of the two in the nominal LISA configuration. (In the case of spherical test masses, the constant angle between the velocity direction and direction to other spacecraft removes some surface irregularity errors that would affect a triangular formation with spherical test masses.) The constant angle between the velocity direction and direction to other spacecraft means that the point-ahead and point-back angles are constant. (These angles arise from the need to point lasers to hit the location the other spacecraft will be at after a one-way light time.) The linear combination needs less spacecraft propulsion to reach the desired orbits (for a given location relative to Earth and arm length L) since no plane change is needed. No laser metrology is needed for the proof masses relative to the optical benches at the outlying spacecraft (e.g., data we have denoted z_{12} , z_{32} , at spacecraft 2). (At the center spacecraft (spacecraft 1) the intra-spacecraft metrology data denoted z_{21} (or $-z_{31}$) in the TDI equations must still be taken to eliminate phase fluctuations due to vibrations of the optical benches.)

The procedure for elimination of USO noise introduces considerable complication in the LISA system, inasmuch as additional calibrating data must be taken and exchanged if present generation flight-qualified USO's are used. The USO noise enters the TDI combinations multiplied by the internally generated fringe tracking frequencies ω_{ij} that are required when there are inter-spacecraft Doppler drifts and/or offsets of the laser center frequencies. Control of relative spacecraft motions will automatically be achieved by spacing SyZyGy along a single nearly-circular solar orbit, and laser frequency offsets can be limited either by atomic line stabilization or by use of optical transponders [9]. Thus no wide-band fringe tracking data, USO calibration, and associated modulation of the laser beams are needed by SyZyGy, as will be shown in the next section.

In the linear case the gravitational wave antenna patterns of all the TDI data combination become simple and identical (so-called 6-pulse response). We give this pattern explicitly in Section 2. The pattern will of course sweep over the sky as the spacecraft move along their common orbit. We apply the TDI data combinations in the linear geometry, and show how the only data combinations that are sensitive to GWs have the same signal response as the combination X, the unequal arm Michelson interferometer. We introduce two new independent combinations that have no gravitational wave response, denoted ρ and σ ; they can be used to monitor system noises. All previously identified laser-noise-free data combinations are given in terms of X, ρ and σ .

In Section 3 we derive the SyZyGy response functions for the remaining secondary noises: optical bench vibration, proof mass residual accelerations, optical path noises at the photodetector readouts, and USO noise entering in to the heterodyne measurements. In Section 4 we give numerical calculations and plots for these. Averaging over the sky and polarization states, we finally plot the resulting gravitational wave sensitivities to incident sine waves, using one year observation time and several SyZyGy scale sizes. Compared with an equilateral LISA, a SyZyGy of the same scale has similar best and high-frequency gravitational wave sensitivities. However the sensitivity is poorer at low-frequencies, suggesting applicability in the frequency band between those of ground-based interferometers and LISA.

II. ANALYSIS

We use the notation and results of [2, 5]. A straight interferometric array, shown in Figure 1, is obtained by opening up the general triangular configuration of Figure 2 of reference [2], so that spacecraft 1 is in the center and $L_1 = L_2 + L_3$. We denote the linear orientation by the unit vector $\hat{n} = \hat{n}_1 = -\hat{n}_2 = -\hat{n}_3$. On each of the spacecraft the usual pair of optical benches can be fused into one, so we put $\vec{V}_i^* = \vec{V}_i$. At the outlying spacecraft 2 and 3 both incoming beams will be referenced to the same face of a single proof mass. At the center spacecraft, number 1, TDI will require use of two opposed faces of a single proof mass. For all we will have $\vec{v}_i^* = \vec{v}_i$ and the optical bench and proof mass motion components that enter are $\vec{V}_i \cdot \hat{n}$ and $\vec{v}_i \cdot \hat{n}$, respectively. The intra-spacecraft data z_{ij} at the three proof masses consequently satisfy $z_{32} = z_{12}$, $z_{13} = z_{23}$, and $z_{21} = -z_{31}$. The first four of these then drop out of all the laser-noise-free data combinations and so in fact need not be recorded. The last two remain, and so one of them needs to be taken, which amounts to implementing precision laser metrology solely on the central spacecraft.

When all these specializations are substituted into the data combinations derived for general configurations, it is found that they degenerate into essentially one, six pulse, response to incident gravitational waves (we use X) plus two others that respond only to system noises (we call them ρ and σ). All three will further be USO-noise-free to acceptable limits if the laser frequency offsets are limited by Doppler effects due to relatively stable spacecraft motion in the SyZyGy linear orbital configuration.

Specifically, the new laser-noise-free combinations we use for a linear array are

$$X = y_{32;3'22'} - y_{23;233'} + y_{31;22'} - y_{21;33'} + y_{23;2} - y_{32;3'} + y_{21} - y_{31} - z_{21} - z_{21;11'} + z_{21;22'} + z_{21;33'}$$

$$\rho = y_{13} - y_{23} - y_{31;2'} + z_{31;2'}$$

$$\sigma = y_{12} - y_{32} - y_{21;3} + z_{21;3}$$

where a semicolon subscripted index i is understood to mean not only time delay by L_i but also multiplication by the Doppler factor $(1 - \dot{L}_i/c)$, which in this case can be dropped². Note how interchanging indices 2 and 3 takes ρ and σ into one another.

² In more general work [3, 4] the TDI combinations have been modified for the case of a moving array where time delays such as L_1 from spacecraft 2 to spacecraft 3 differ from L'_1 from spacecraft 3 to spacecraft 2, etc. One then has for SyZyGy $L_1 = L'_2 + L'_3$ and $L'_1 = L_2 + L_3$. The primes on the time-delay subscripts in these equations for X , ρ , and σ have been inserted for that case, but we do not insist on that generality in any of the subsequent discussion.

The previously used TDI basic combinations in this linear array limit (and without motion) are given in terms of X , ρ , and σ :

$$\alpha = X + \rho_{;2} - \sigma_{;3}$$

$$\beta = -\sigma + \rho_{;1}$$

$$\gamma = \rho - \sigma_{;1}$$

$$\zeta = \rho_{;3} - \sigma_{;2}$$

Proof mass and shot noise response functions for X and ρ (or σ) are given in Section 4.

III. NOISE TRANSFER FUNCTIONS

The proof-mass-plus-optical-bench assembly and laser beam paths for the central spacecraft is show schematically in Figure 2. The photodetectors that generate the time series y_{21} , y_{31} , and z_{21} are as indicated. The outgoing light from spacecraft 1 to spacecraft 2 is routed from laser 1 on the optical bench using mirrors and beam splitters; this beam does not interact with spacecraft 1's proof mass. Conversely, the incoming light beam from spacecraft 2 is first bounced off the nearest face of proof mass 1 before being reflected onto the photodetector, where it is mixed with light from laser 1. This time series is y_{31} in Figure 2. Analogously, the time series y_{21} is producted from light transmitted to and received from spacecraft 3. The relative motion of the optical bench and proof mass is monitored by measuring the Doppler shift of the light coming directly from the laser with the light bounced off one face of the proof mass (this forms the metrology time series $z_{21} = -z_{31}$, as indicated in Figure 2).

Figure 3 shows the light paths on spacecraft 3. The outgoing light beam from spacecraft 3 to spacecraft 1 and 2 is routed from laser 3 on spacecraft 3's optical bench using mirrors and beamsplitters; again, this beam does not interact with spacecraft 3's proof mass. The two incoming light beams from spacecraft 1 and 2 are bounced off the proof mass before being reflected onto photodetectors where they are mixed with light from laser 3. The time series produced are y_{23} and y_{13} , as shown. The configuration at spacecraft 2 is analogous, producing y_{12} and y_{32} . y_{13} and y_{12} enter α , ρ , and σ , but not X .

The equations for the noises entering the y_{ij} and $z_{21} = -z_{31}$ can be developed from Figures 2 and 3. Consider first the situation on spacecraft 1. The photodetector producing y_{31} (moving with velocity V_1) reads that time series by mixing the beam originating from

the optical bench on spacecraft 2 sent in direction $\hat{n}_3 = -\hat{n}$ (laser noise C_2 and optical bench motion \vec{V}_2 , delayed by propagation along L_3), after one bounce off the proof mass (\vec{v}_1), with the local laser light (C_1). The time series y_{21} is produced analogously. The z_{21} measurement is from light originating at the laser (C_1, \vec{V}_1), bounced off the left face of the proof mass (\vec{v}_1), and mixed with the direct laser light (C_1). For spacecraft 2 and 3, y_{32} , and y_{23} are produced similarly. Metrology time series on the end spacecraft are not required.

We will see below that there is no gravitational wave contribution to the data ρ and σ . It may however be desirable to measure these to monitor system responses and secondary noises. In that case we would also need the data y_{12} and y_{13} .

The y_{ij} and the required metrology time series z_{21} ($= -z_{31}$), including gravitational wave signals and shot noises, can be developed from the diagrams and by consulting the Appendix of Tinto *et al.* [5]:

$$\begin{aligned} \nu_0 y_{21} = & \nu_3(1 + C_{3,2} - \hat{n}_2 \cdot \vec{V}_{3,2})\Delta_2(1 + 2\hat{n}_2 \cdot \vec{v}_1 - \hat{n}_2 \cdot \vec{V}_1) \\ & - \nu_1(1 + C_1) - \omega_{21}(1 + Q_1) + \nu_0 y_{21}^{gw} + \nu_0 y_{21}^{opt. path} \end{aligned} \quad (1)$$

$$\begin{aligned} \nu_0 y_{31} = & \nu_2(1 + C_{2,3'} + \hat{n}_3 \cdot \vec{V}_{2,3'})\Delta_3(1 - 2\hat{n}_3 \cdot \vec{v}_1 + \hat{n}_3 \cdot \vec{V}_1) \\ & - \nu_1(1 + C_1) - \omega_{31}(1 + Q_1) + \nu_0 y_{31}^{gw} + \nu_0 y_{31}^{opt. path} \end{aligned} \quad (2)$$

$$\nu_0 z_{21} = 2\nu_1 \hat{n}_3 \cdot (\vec{v}_1 - \vec{V}_1) = -\nu_0 z_{31} \quad (3)$$

where $\Delta_i = (1 - \dot{L}_i/c)$. Other y_{ij} are determined from equations (1) and (2) by cyclic index permutation.

Inserting these into the equations for α and ρ we can verify that the laser phase/frequency noises cancel and that the optical bench noises and USO noises are, for reasonable parameters of SyZyGy, suppressed to acceptable levels. Noting that $\Delta_2\Delta_3 \simeq \Delta_1$ and $\omega_{23} - \omega_{13} = \nu_1\Delta_2 - \nu_2\Delta_1 \simeq -\omega_{31}$, and its permutations, we then obtain the following responses to proof-mass, USO, and optical path noises in X and ρ or σ .

$$\begin{aligned}
X^{noise} &= X^{proof\ mass} + X^{USO} + X^{opt.\ path} \\
&= -2(\nu_1/\nu_0)(\hat{n} \cdot \vec{v}_1 - \hat{n} \cdot \vec{v}_{1;11}) + 2(\nu_2/\nu_0)(\hat{n} \cdot \vec{v}_{2;3} - \hat{n} \cdot \vec{v}_{2;322}) \\
&\quad + 2(\nu_3/\nu_0)(\hat{n} \cdot \vec{v}_{3;2} - \hat{n} \cdot \vec{v}_{3;233}) \\
&\quad - 2(\omega_{31}/\nu_0)(\hat{n} \cdot \vec{v}_1 - \hat{n} \cdot \vec{v}_{1;22}) - 2(\omega_{21}/\nu_0)(\hat{n} \cdot \vec{v}_1 - \hat{n} \cdot \vec{v}_{1;33}) \\
&\quad + 2(\omega_{32}/\nu_0)(\hat{n} \cdot \vec{v}_{2;3} - \hat{n} \cdot \vec{v}_{2;322}) + 2(\omega_{23}/\nu_0)(\hat{n} \cdot \vec{v}_{3;2} - \hat{n} \cdot \vec{v}_{3;233}) \\
&\quad + (\omega_{31}/\nu_0)(Q_1 - Q_{1;22}) - (\omega_{21}/\nu_0)(Q_1 - Q_{1;33}) \\
&\quad + (\omega_{32}/\nu_0)(Q_{2;3} - Q_{2;12}) - (\omega_{23}/\nu_0)(Q_{3;2} - Q_{3;13}) \\
&\quad + y_{32;322}^{opt.\ path} - y_{23;233}^{opt.\ path} + y_{31;22}^{opt.\ path} - y_{21;33}^{opt.\ path} \\
&\quad + y_{23;2}^{opt.\ path} - y_{32;3}^{opt.\ path} + y_{21}^{opt.\ path} - y_{31}^{opt.\ path}
\end{aligned} \tag{4}$$

$$\begin{aligned}
\rho^{noise} &= \rho^{proof\ mass} + \rho^{USO} + \rho^{opt.\ path} \\
&= 2(\omega_{31}/\nu_0)[\hat{n} \cdot \vec{v}_3 - \hat{n} \cdot \vec{v}_{1;2}] \\
&\quad - (\omega_{31}/\nu_0)(Q_3 - Q_{1;2}) \\
&\quad + y_{31}^{opt.\ path} - y_{23}^{opt.\ path} - \Delta_2 y_{31;2}^{opt.\ path}
\end{aligned} \tag{5}$$

Under the assumptions that all noises are independent, that optical path noise only depends on the distance between spacecraft pairs, and that for proof mass noise in X terms with coefficients ω_{ij}/ν_0 are negligible compared with terms having coefficients of order unity, the spectra of the noises in X and ρ are:

$$\begin{aligned}
S_X(f) &= S_X^{proof\ mass} + S_X^{USO} + S_X^{opt.\ path} \\
&\simeq 16[\sin^2(2\pi f L_1) + \sin^2(2\pi f L_2) + \sin^2(2\pi f L_3)]S^{proof\ mass} \\
&\quad + [4(\omega_{32}^2 + \omega_{31}^2)/\nu_0^2 \sin^2(2\pi f L_2) + 4(\omega_{23}^2 + \omega_{21}^2)/\nu_0^2 \sin^2(2\pi f L_3) \\
&\quad - 8(\omega_{21}\omega_{31})/\nu_0^2 \cos(2\pi f(L_2 - L_3)) \sin(2\pi f L_2) \sin(2\pi f L_3)]S^{USO} \\
&\quad + 8\sin^2(2\pi f L_3)S_{23}^{opt.\ path} + 8\sin^2(2\pi f L_2)S_{31}^{opt.\ path}
\end{aligned} \tag{6}$$

$$\begin{aligned}
S_\rho(f) &= S_\rho^{proof\ mass} + S_\rho^{USO} + S_\rho^{opt. path} \\
&= 8(\omega_{31}^2/\nu_0^2)S^{proof\ mass} + 2(\omega_{31}^2/\nu_0^2)S^{USO} \\
&\quad + S_{13}^{opt. path} + S_{23}^{opt. path} + S_{31}^{opt. path}
\end{aligned} \tag{7}$$

IV. GRAVITATIONAL WAVE SIGNAL RESPONSE

The gravitational wave signal response in TDI combination X was given for a general triangle in equation (24) of reference [1]. The SyZyGy configuration leads to considerable simplification. Taking the limit as the general triangle is distorted to a line, using $L_1 = L_2 + L_3$, and remembering that $\hat{n} = \hat{n}_1 = -\hat{n}_2 = -\hat{n}_3$, we obtain a six pulse response with a single amplitude depending only on the orientation of the linear array with respect to the source direction:

$$\begin{aligned}
X^{gw} &= 2 \hat{k} \cdot \hat{n} [\Psi(t - L_2 + L_2 \hat{k} \cdot \hat{n}) - \Psi(t) \\
&\quad + \Psi(t - L_3 - L_3 \hat{k} \cdot \hat{n}) - \Psi(t - L_1 - L_2 - L_3 \hat{k} \cdot \hat{n}) \\
&\quad + \Psi(t - L_1 - L_2 - L_3) - \Psi(t - L_1 - L_3 + L_2 \hat{k} \cdot \hat{n})]
\end{aligned} \tag{8}$$

where \hat{k} is the gravitational wavevector and Ψ is (see [11, 12])

$$\Psi(t) = \frac{1}{2} \frac{\hat{n} \cdot \mathbf{h}(t) \cdot \hat{n}}{1 - (\hat{k} \cdot \hat{n})^2} \tag{9}$$

for a transverse traceless gravitational wave $\mathbf{h}(t) = [h_+(t) \mathbf{e}_+ + h_\times(t) \mathbf{e}_\times]$. Note that X^{gw} is zero for waves incident at right angles to SyZyGy ($\hat{k} \cdot \hat{n} = 0$) and for waves parallel or antiparallel with it ($\hat{k} \cdot \hat{n} = +1$ or -1).

Although SyZyGy will not operate exclusively in the long-wavelength limit (LWL - $2\pi f L_1 \ll 1$), analytical results at low-frequency are useful. Expanding the gravitational wave \mathbf{h} in series of spatial derivatives and substituting into X^{gw} gives

$$X^{gw} \rightarrow -\mu L_1 L_2 L_3 \hat{n} \cdot \mathbf{h}''' \cdot \hat{n} \tag{10}$$

where $\mu = \hat{k} \cdot \hat{n}$.

V. GRAVITATIONAL WAVE SENSITIVITY

Figure 4 shows root-mean-square X^{gw} , averaged over polarization and the celestial sphere [1, 2], for 3 armlength cases: (a) $L_2 = L_3 = 16$ light-seconds, (b) $L_2 = 7, L_3 = 9$ light-seconds, and (c) $L_2 = 0.7, L_3 = 0.9$ light-seconds. Notice that in the LWL $X^{gw} \propto f^3$, as expected from the LWL expansion above.

Figure 5 shows the noise power spectra of X, using proof mass and optical path noises only, for the same armlength cases as Figure 4. (USO can be made negligible compared to the other noises through trajectory control: if the maximum relative speed is kept below 0.2 m/sec—either through trajectory design or by periodically using thrusters to adjust the maximum Doppler shift—USO noise will not contribute.) In constructing the curves in Figure 5 we used the same instrumental spectra as in our previous sensitivity analysis of LISA. The proof masses were assumed to have independent noises with one-sided acceleration spectral density $3 \times 10^{-15} \text{ m sec}^{-2} \text{ Hz}^{-1/2}$ (this corresponds [2, 5] to a one-sided spectral density of relative Doppler $S_y^{proof\ mass} = 2.5 \times 10^{-48} [\text{f}/1 \text{ Hz}]^{-2} \text{ Hz}^{-1}$). Following previous practice [2, 5], we approximately account for all optical path noise, including beam pointing noise (see [2] and reference [13], Table 4.1), on a given link with $S_y^{optical\ path} = 1.8 \times 10^{-37} [\text{f}/1 \text{ Hz}]^2 (\text{r}/5 \times 10^9 \text{ m})^2 \text{ Hz}^{-1}$, where r is the distance between spacecraft on that link. The aggregate noise spectrum plotted uses these spectra and the X transfer function (see equations 4 and 6).

The GW sensitivity is proportional to the ratio of the rms noise to the rms signal. Figure 6 shows the sensitivity of SyZyGy expressed in the conventional way: sensitivity to isotropic sinusoidal gravitational radiation ($\text{SNR} = 5$ in one-year integration), $5[S_X(f) B]^{1/2}/\text{rms } X^{gw}(f)$, where $B = 1$ cycle/year, for the three armlength cases of Figures 4 and 5.

VI. DISCUSSION

SyZyGy offers some technical simplifications compared with other configurations. Each spacecraft utilizes a single proof mass, and laser metrology is needed only at the central spacecraft. Because of the low relative velocities afforded by the SyZyGy orbit, the complexity needed for USO noise elimination (laser beam modulation and additional calibration data) [5] is not required by SyZyGy. The angles between the arms are much more constant in time than for LISA, so time-dependent articulation of the telescopes is not required.

SyZyGy combinations ρ and σ have exactly zero GW response and can thus be used with no ambiguity to assess on-orbit noise performance. Deliberate resetting of arm lengths (i.e., different arm lengths in different portions of a long mission) is easier in a linear configuration and could be used to tune SyZyGy's best response over its mission lifetime.

For similar scale size, SyZyGy achieves both its best sensitivity and high-frequency performance comparable to those of LISA; at low-frequencies, SyZyGy has poorer sensitivity. This suggests applicability in the Fourier band between those of LISA and ground-based interferometers. Little attention has been given to the $\sim 0.1 - 10$ Hz band. The work that has been done suggests it to be relatively free of foreground (astrophysical) GW sources [14]. A SyZyGy with $L_1 \sim 1$ light-second could test this with 5σ sensitivity $\simeq 2.5 \times 10^{-23}$ in a year's integration.

Acknowledgments

This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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FIG. 1. SyZyGy configuration. Unit vectors \hat{n}_i point between spacecraft pairs with the indicated orientations. The arrows on the L_i indicate the sense of the un-primed light times. Each spacecraft has one optical bench and one proof mass.

FIG. 2. Signal routing and readouts on the central spacecraft, 1.

FIG. 3. Signal routing and readouts on spacecraft 3, one of the end spacecraft.

FIG. 4. RMS gravitational wave response of SyZyGy, TDI combination X, as a function of Fourier frequency for three cases: $L_2 = L_3 = 16$ light-seconds; $L_2 = 7, L_3 = 9$ light seconds; and $L_2 = 0.7, L_3 = 0.9$ light seconds.

FIG. 5. One-sided noise spectra of X as a function of Fourier frequency. One-sided spectra of proof mass noise and optical path noise are taken to be the same as we have used in previous calculation of LISA sensitivity [2, 5]: respectively, $3 \times 10^{-15} \text{ m sec}^{-2}\text{Hz}^{-1/2}$ and $20 \times 10^{-12} \times (r/5 \times 10^9 \text{ m}) \text{ m Hz}^{-1/2}$, where r is the distance between spacecraft. Center frequencies of the lasers are assumed to be equal and a maximum Doppler shift (longest arm case) of 0.2 m/sec is assumed. Solid lines: spectra of X including shot and proof mass noises. Dashed lines: contribution to X noise spectrum from an uncalibrated USO having one-sided spectrum of fractional frequency fluctuations $8 \times 10^{-27}/[f/1\text{Hz}]$ (see text).

FIG. 6. Gravitational wave sensitivity of X for SyZyGy (SNR = 5 in a one year integration, sky averaged, sinusoidal signals). Arm lengths, gravitational wave responses, and noise spectra are as in Figures 4-5.











